Berlin University of Technology

Computer Vision & Remote Sensing

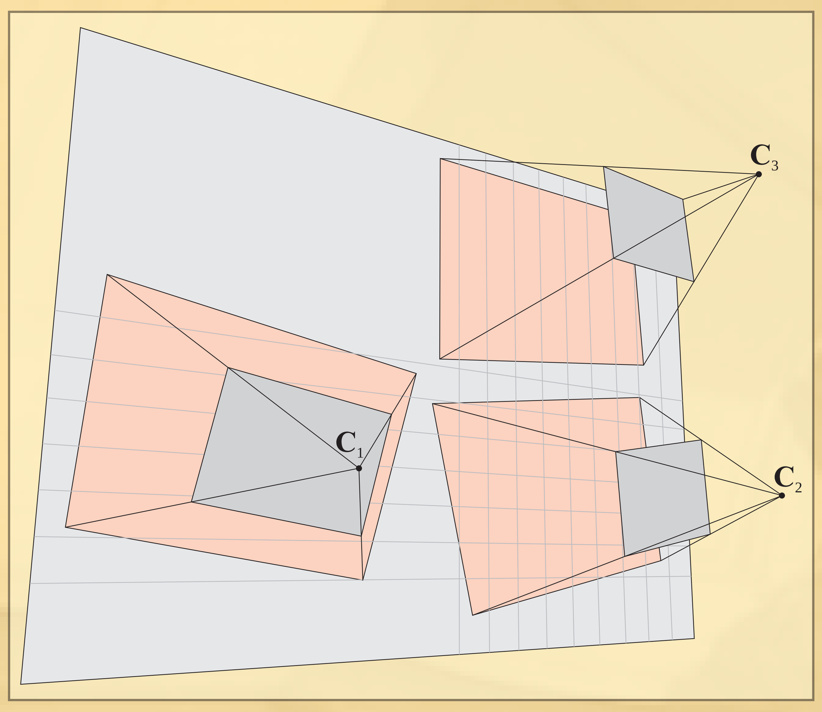
Trinocular Rectification for Various Camera Setups

Matthias Heinrichs and Volker Rodehorst

Introduction

What is Trinocular Rectification?

- A geometric transformation of an image triplet of the same scene that makes epipolar lines parallel to the image axes.
- Collinear camera setups can be rectified by chaining binocular rectifications.
- General camera configurations are more complex.



Advantages

- Known relative orientation reduces the matching search space to epipolar lines.
- A match candidate in two images can be verified by its position in the third image.
- In rectified images the epipolar lines correspond to the image raster, which allows very efficient matrix processing.
- The horizontal and vertical disparities of any image point can be made identical.

Applications

Any image matching approach using three (uncalibrated) images.

Proposed Method

A linear planar rectification method coupled with reasonable constraints for the six remaining degrees of freedom (DoF).

Requirements

- The cameras are not collinear.
- Each projection center $\mathbf{C}_i$ must lie outside all other images.
- At least six corresponding image points are necessary.

Functional Model

- Each projective transformation (homography) has eight free parameters. All three homographies have 24 DoF.
- The uncalibrated relative orientation (fundamental matrix) has seven significant parameters. All three fundamental matrices must be compatible and fulfill three additional linear constraints:

$$\mathbf{e}_{hv}^{T}\mathbf{F}_{hb}\mathbf{e}_{bv} \quad \mathbf{e}_{vb}^{T}\mathbf{F}_{vh}\mathbf{e}_{hb} \quad \mathbf{e}_{vh}^{T}\mathbf{F}_{vb}\mathbf{e}_{bh} \quad 0$$

Overall, the fundamental matrices have 18 linearly independent parameters.

Relative Orientation

Trifocal Tensor

The trifocal tensor for the three images is computed by using the minimal 6-point-algorithm. The three compatible fundamental matrices are extracted from this tensor .

Fundamental Matrices

The desired fundamental matrices of the rectified images are given by:

$$\tilde{\mathbf{F}}_{bh} \begin{matrix}0&0&0\\0&0&1\\0&1&0\end{matrix} ,\; \tilde{\mathbf{F}}_{bv} \begin{matrix}0&0&1\\0&0&0\\1&0&0\end{matrix} ,\; \tilde{\mathbf{F}}_{hv} \begin{matrix}0&0&1\\0&0&1\\1&1&0\end{matrix}$$

Camera Setup and Alignment

Desired Camera Setup

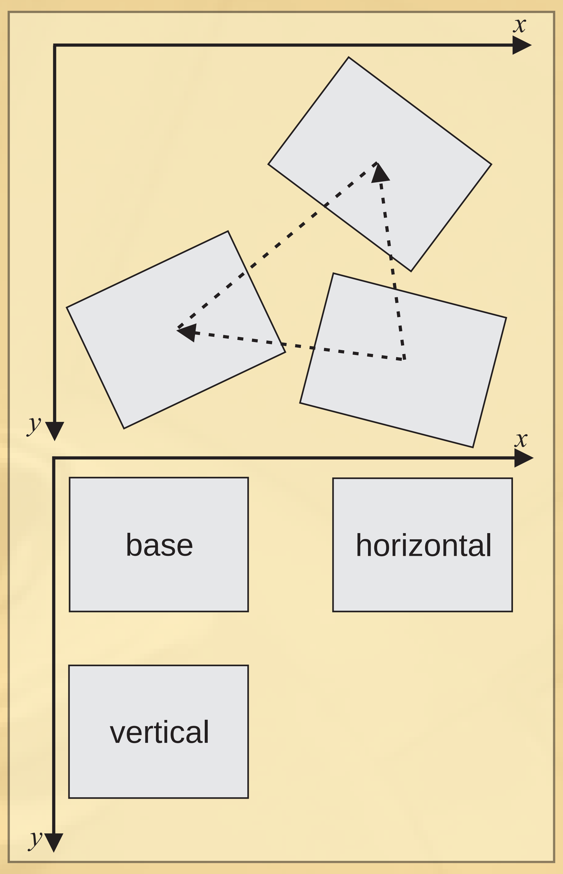
The positions of images in a given triplet are classified as b (base), h (horizontal) and v (vertical).

- Epipolar lines of b and h correspond with rows.
- Epipolar lines of b and v correspond with columns.
- Epipolar lines of h and v have slope minus unity.

The trifocal tensor of the unrectified images gives three projection matrices, from which the angle between the camera centers in the x/y-plane can be derived:

- The highest angle is between b and v.
- The lowest angle is between b and h.

After determining the basic orientation, flipping along the x- and/or y-axis might be necessary.



System Overview

Linear Rectification Homographies

$$\mathbf{H}_h \quad \begin{matrix}1&0&s_1&s_3\\0&1&s_2&\\0&0&1&\end{matrix} \quad \begin{matrix}m_x\;{}_3&0&0\\0&m_y\;{}_3&0\\0&0&1\end{matrix} \quad \begin{matrix}1\,1&{}_1\,F_{33}^{bv}(\;{}_2\;\;1)\\0\;\;{}_1&0\\0\;\;0&1\end{matrix} \quad \begin{matrix}w_{h1}(F_{33}^{bv}\;\;F_{33}^{bh})\;\;F_{13}^{bh}\\w_{h1}F_{33}^{bh}\;\;F_{13}^{bh}\\w_{h1}\end{matrix} \quad \begin{matrix}F_{31}^{hv}\;\;w_{h2}(F_{33}^{bv}\;\;F_{33}^{bh})\;\;F_{23}^{bh}\\w_{h2}F_{33}^{bh}\;\;F_{23}^{bh}\\w_{h2}\end{matrix} \quad \begin{matrix}F_{32}^{hv}\;\;F_{33}^{bv}\;\;F_{33}^{hv}\\0\\1\end{matrix}$$

$$\mathbf{H}_v \quad \begin{matrix}1&0&s_1&\\0&1&s_2&s_3\\0&0&1&\end{matrix} \quad \begin{matrix}m_x\;{}_3&0&0\\0&m_y\;{}_3&0\\0&0&1\end{matrix} \quad \begin{matrix}{}_2&0&0\\1\;\;{}_2&1\;F_{33}^{bv}(\;{}_2\;\;1)\\0&0&1\end{matrix} \quad \begin{matrix}F_{13}^{bv}\\w_{v1}(F_{33}^{bv}\;\;F_{33}^{hv})\;\;F_{13}^{hv}\\w_{v1}\end{matrix} \quad \begin{matrix}F_{23}^{bv}\\F_{13}^{bv}\;\;w_{v2}(F_{33}^{bv}\;\;F_{33}^{hv})\;\;F_{23}^{hv}\\w_{v2}\end{matrix} \quad \begin{matrix}F_{33}^{bv}\\F_{23}^{bv}\;\;0\\1\end{matrix}$$

$$\mathbf{H}_b \quad \begin{matrix}1&0&s_1\\0&1&s_2\\0&0&1\end{matrix} \quad \begin{matrix}m_x\;{}_3&0&0\\0&m_y\;{}_3&0\\0&0&1\end{matrix} \quad \begin{matrix}{}_2&0&0\\0&{}_1&0\\0&0&1\end{matrix} \quad \begin{matrix}w_{b1}F_{33}^{bv}\;\;F_{31}^{bv}&w_{b2}F_{33}^{bv}\;\;F_{32}^{bv}&0\\F_{31}^{bh}&F_{32}^{bh}&F_{33}^{bh}\\w_{b1}&w_{b2}&1\end{matrix}$$

with $\mathbf{w}_b \quad \mathbf{e}_{bh} \quad \mathbf{e}_{bv}$, $\mathbf{w}_h \quad \mathbf{e}_{hb} \quad \mathbf{e}_{hv}$ and $\mathbf{w}_v \quad \mathbf{e}_{vb} \quad \mathbf{e}_{vh}$.

Geometric Interpretation of the six DoF

1. s_1 is the global translation of all images in the x-direction.
2. s_2 defines the global shift value in the y-direction.
3. s_3 is the translation in x of h and in y of v relative to b.
4. α_1 is scale of the y-component of b and h, which affects the shearing in the y-direction of v.
5. α_2 is scale of the x-component of b and v, which affects the shearing in the x-direction of h.
6. α_3 is a global scaling factor, used to keep the images at a suitable resolution.

Convenience Parameters

1. m_x is a mirroring factor in the x-direction of all images.
2. m_y defines the mirroring factor in the y-direction.

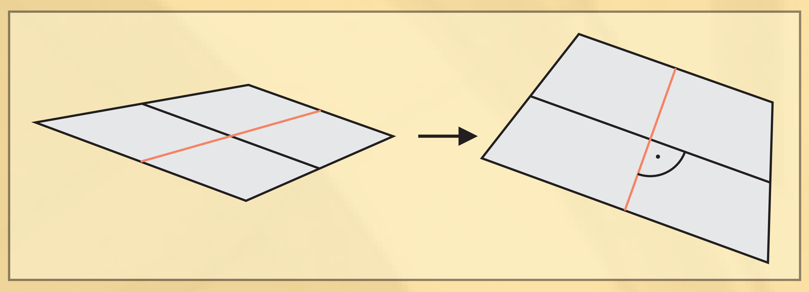
Proposed Algorithm

1. Compute compatible fundamental matrices.
2. Classify image positions as b, h and v.
3. Estimate basic rectification homographies.
4. Find shearing values α_1 and α_2.
5. Compute global scale α_3 to maintain the image size.
6. Correct potential mirroring using m_x and m_y.
7. Find suitable offset values s_1, s_2 and s_3 to place the images around the origin.
8. Resample images.

Experimental Results

Shearing Correction

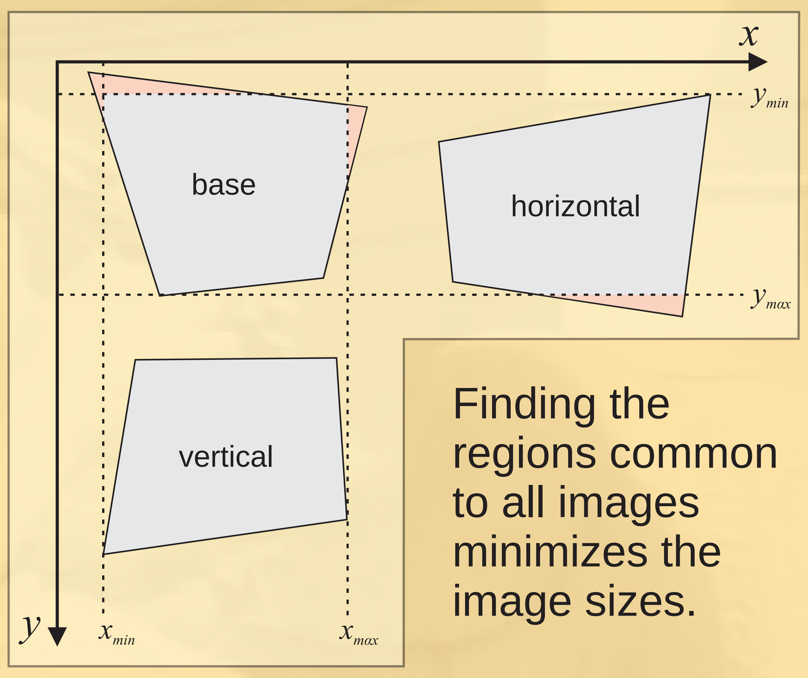
Keep vectors in the middle of the image perpendicular to minimize shearing in the rectified images:



- Calculate two vectors $\mathbf{x}$ and $\mathbf{y}$ along the original image halves.
- Apply the homographies with $\alpha_1 = 1$ and $\alpha_2 = 1$ to get the vectors $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$.
- To make $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ perpendicular again, solve a cubic equation for h and v:

$$(\mathbf{S}_h\tilde{\mathbf{x}})^{\mathrm{T}}(\mathbf{S}_h\tilde{\mathbf{y}}) \quad 0 \;\text{ with }\; \mathbf{S}_h \quad \begin{matrix}1\,1&{}_1&0\\0&{}_1&0\\0&0&1\end{matrix}$$

Common Region Cropping



Finding the regions common to all images minimizes the image sizes.

Quantitative Evaluation

Test Input Data

- Original image size of 1024 768 pixels.
- Obtained with a hand held digital camera.
- 24 manually determined image point correspondences which are well distributed over the scene.
- The estimated trifocal tensor was refined using non-linear LM optimization.

Evaluation

After rectification the following image distance errors (in pixels) remain:

Distance	b ? h	b ? v	h ? v	Total
Mean error	0.378	0.285	0.573	0.398
Variance	0.374	0.163	0.688	0.393

Image sets of three different scenes are passed in several different orderings to check the camera alignment. Accurate rectifications were obtained in all cases.

Conclusions

- A linear method for trinocular rectification of (uncalibrated) images in closed form with six DoF was proposed.
- Proper constraints and geometric interpretation of the remaining DoF are given.
- Automated image alignment allows more convenient image acquisition.
- Mirror correction permits a broad range of camera setups.
- Rectification quality depends on the robust estimation of the relative orientation (trifocal tensor).

Acknowledgements

This work was partly founded by the german research foundation DFG.

Information: www.cv.tu-berlin.de Contact: Matthias Heinrichs ☎ +49 30 314 73721 +49 30 314 21104 matzeh@cs.tu-berlin.de